Theoretical Considerations for a Portable and Reusable Electromagnetic Pulse Weapon powered solely by Electricity

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EMP is most effective when the target is blown up or fries. This occurs when the e.m.f. induced in the target is high.

By Faraday's Law: $\varepsilon \propto \frac{d\Phi}{dt}$ Φ is magnetic flux.

$\therefore$ Transmitter must produce large $\frac{d\Phi}{dt}$ The more the merrier.

$\frac{d\Phi}{dt} = \frac{d(AB)}{dt} = A\frac{dB}{dt} + B\frac{dA}{dt}$ (but area is constant and thus $B\frac{dA}{dt} = 0$)

$= A\frac{dB}{dt}$ ($A$is the area through which B passes. B is the magnetic field strength)

This means a high $\frac{dB}{dt}$ is needed.

By Biot-Savart's Law:

$dB = \frac{\mu_0}{4\pi}\frac{Idl \times \hat{r}}{r^2}$

which is complicated, but turns out it simplifies to:

$B = \frac{\mu_0 I}{2\pi r}$ for straight wires, and

$B = \frac{\mu_0 NI}{l}$ for solenoids, where N is the number of turns of wire, I the current through it, l the length of the solenoid, r is the radius away from the wire which you're measuring the B-field.

Since we want to max $\frac{dB}{dt}$, we should use a solenoid, and not a straight wire, because solenoids concentrate magnetic fields (as evidenced by electromagnets being made from solenoids). From the formula, it appears that having a high N and I, while having a small l, would give the high B needed.

However, a solenoid carries with it inductance L which is governed by:

$V = L\frac{dI}{dt}$ where V is the voltage across the inductor.

Rearranging gives:

$\frac{V}{L} = \frac{dI}{dt}$

Which means the smaller the inductance, the better, and the higher the voltage, the better.

The formula for calculating inductance in a solenoid is:

$$L \approx \frac{\mu_0 \pi N^2 r^2}{l} K$$

(where $K \to 1$ as $\frac{l}{2r} \to \infty$, and for $\frac{l}{2r} = 10$, K = 0.95, otherwise, K = 0.06 for $\frac{d}{l} = 50$, 0.08 for $\frac{d}{l} = 40$, 0.1 for $\frac{d}{l} = 25$, 0.15 for $\frac{d}{l} = 15$, 0.2 for $\frac{d}{l} = 10$, 0.32 for $\frac{d}{l} = 5$, 0.43 for $\frac{d}{l} = 3$, 0.52 for $\frac{d}{l} = 2$)

where N is the number of turns, r is the radius, l is the length, d is the diameter, all of the solenoid.

While the formula for calculating inductance in a single circular loop (a special case of a solenoid) is:

$$L \approx \mu_0 r \left(\ln \frac{8r}{a} - 1.75 \right)$$

where r is radius of loop and a is the wire's radius.

So, substituting in the solenoid formula for B,

$$A \frac{dB}{dt} = A \frac{\mu_0 N}{l} \frac{dI}{dt}$$

And as we know,

$$\frac{dI}{dt}_{\max} = \frac{V}{L}_{\max}$$ but $\frac{V}{L}$ doesn't change, so it's just

$$\frac{dI}{dt}_{\max} = \frac{V}{L}$$

Substituting in:

$$\frac{d\Phi}{dt}_{\max} = A \frac{dB}{dt}_{\max} = A \frac{\mu_0 \pi r^2 NV}{lL}$$

where A is the area of the coil, r is the radius, N is the number of turns, V is the voltage across the coil, L is the inductance, and l is the length.

We can substitute in for L using the solenoid formula (above) making:

$$\frac{d\Phi}{dt}_{\max} = \frac{\mu_0 \pi r^2 NVl}{\mu_0 \pi N^2 r^2 Kl} = \frac{V}{KN}$$

From here, we realise that N should be as small as possible. The smallest is 1, which means the solenoid should be a single loop.

However, using a single loop changes the starting formula a little.
Recalling Biot-Savart's law:

$$dB = \frac{\mu_0}{4\pi}\frac{Idl \times \hat{r}}{r^2}$$ but we don't need direction here, so get rid of that unit vector.

$$\int dB = \int \frac{\mu_0}{4\pi}\frac{Idl}{r^2}$$

$$= \frac{\mu_0 I}{4\pi r^2}\int dl$$

Since we're using a loop, and calculating the B-field directly in the centre, *dl* is a infinitsimal portion of that loop, which when integrated, gives the entire loop, or other words, the circumference.

$$\therefore B = \frac{\mu_0 I}{4\pi r^2} 2\pi r$$

$$= \frac{\mu_0 I}{2r}$$

differentiating both sides with respect to *t*,

$$\frac{dB}{dt} = \frac{\mu_0}{2r}\frac{dI}{dt}$$

As earlier mentioned,

$$\frac{dI}{dt} = \frac{V}{L}$$

Substituting both in gives:

$$\frac{d\Phi}{dt}_{\max} = A\frac{dB}{dt}_{\max} = \frac{A\mu_0 V}{2rL}$$

Now, substituting the single loop formula for inductance *L* in:

$$\frac{d\Phi}{dt}_{\max} = \frac{A\mu_0 V}{2r\mu_0 r\left(\ln\frac{8r}{a} - 1.75\right)}$$

$$= \frac{AV}{2r^2\left(\ln\frac{8r}{a} - 1.75\right)}$$

By looking at the result, immediately, 2 values make $\frac{d\Phi}{dt}_{\max}$ become the largest, i.e. infinity. Either

$r = 0$, or

$\ln\frac{8r}{a} - 1.75 = 0$, which solves to $\frac{r}{a} \approx 0.7193$.

Both are impossible to achieve because a loop with a diameter of 0 is not a loop anymore, and a loop with the radius at 0.7193 times of the wire radius means the wire radius is larger than the loop radius, which is also impossible. In fact, at all times, $r > a$ must hold true.

With that in mind, we could substitute 8*r* as 8*a* or more. Because the rest are constants, we could plot a graph of $\frac{d\Phi}{dt}_{\max}$ against *a*. The result is a graph similar to $y = \frac{1}{x^2}$

<u>The conclusion is so far thus, to get a large *a*, while getting a small *r*, and a high *V*, and the propagation method is to use a single wire loop.</u>

Every circuit has 5 components: *V*, *I*, *R*, *L*, and *C*. (voltage, current, resistance, inductance, and capacitance respectively)
We've settled *V*, *I*, and *L*. *R* doesn't seem to have a place.

But $V = IR$, and we've only looked at $\frac{dI}{dt}$ so far. $\frac{dI}{dt} = k$ if we assume it to be constant, thus $I = \int dI = k\int dt = kt$

$I_{\max} = \frac{V}{R_{\max}}$ but again, *V* and *R* doesn't change.
If we substitute that in, then
$$\frac{V}{R} = I_{\max} = kt$$
which tells us that if *R* is lesser, *t* will be longer, and the eventual current will be higher.

In order to minimise *R*, everything in the circuit will have to have minimal resistance, including the power source.
The lowest internal resistant power source is a capacitor. Thus the high current and voltage should be supplied by a capacitor bank.

A capacitor discharges according to the formula:
$$V = V_0 e^{\frac{-t}{RC}}$$
where *V* is the voltage, V_0 is the starting voltage, *t* is time, *R* is the resistance of the circuit, and *C* is the capacitance.

Recall that
$$\frac{V}{L} = \frac{dI}{dt}$$
for the current in an inductor.

Since the voltage is supplied by a capacitor bank, thus we can substitute in the capacitor formula:

$$\frac{V_0 e^{\frac{-t}{RC}}}{L} = \frac{dI}{dt}$$

This again emphasises that we should have low inductance. It also seems to imply that if *R* and/or *C* values are very high, then $\frac{dI}{dt}$ will be always max. This is somewhat true because if *R* and *C* values are high, then the capacitor will always stay close to full charge, and thus will give maximum voltage that will allow the current to rise very rapidly. It may seem awkward, but we will address this shortly.

This formula gives $\frac{dI}{dt}$ as a function of time. If we want to find *I* as a function of time, i.e. *I*(*t*), all we have to do is to integrate it:

$$\int dI = \int \frac{V_0 e^{\frac{-t}{RC}}}{L} dt$$

$$= \frac{V_0}{L} \int e^{\frac{-t}{RC}} \quad \text{which by substitution } x = \frac{-t}{RC} \quad \therefore dt = -RCdx$$

$$= -\frac{V_0 RC}{L} \int e^x dx$$

$$= -\frac{RCV_0 e^x}{L} + k$$

$$= k - \frac{RCV_0 e^{\frac{-t}{RC}}}{L} = I(t)$$

Since at $t = 0$, the current is 0, thus we can find *k*, and the equation simplifies to:

$$I(t) = \frac{RCV_0}{L}\left(1 - e^{\frac{-t}{RC}}\right) \qquad \text{———— (1)}$$

From here, we see that the current rises rapidly at first then slows down as it reaches its maximum. Essentially a $y = k - e^{-x}$ graph (to help you visualise).

Yet, from the looks of it, as $t \to \infty$, $I \to \frac{RCV_0}{L}$ which is non-zero and thus seems absurd. How can a finite capacitor bank sustain the current for an infinite amount of time? Also, as stated earlier, is it ideal that *R* is high? The shortfall of this formula is that it doesn't consider the "finite-ness" of the capacitor bank, i.e. that as its voltage drops, its current also drops. Also, the *R* stated is the resistance in the circuit that slows down the voltage drop. But while *R* does not have effect on the current rise, it limits the length of time the current <u>can</u> rise, limiting the pulse time, and limiting the maximum current. We need another formula to address the shortfalls.

Recall that the voltage of a capacitor for discharge is governed by:

$$V = V_0 e^{\frac{-t}{RC}}$$

and thus this implies that the current will be, by applying Ohm's law:

$$\frac{V}{R} = \frac{V_0}{R} e^{\frac{-t}{RC}} = I(t) \quad \text{———} (2)$$

This is essentially a $y = e^{-x}$ graph (to help you visualise).

We now have two functions of current $I(t)$. We need to combine both to get the "real" result. (2) gives the supply "capability" of the capacitor bank, while (1) is the draw capacity of the inductor. (1) only counts in the voltage of the capacitor bank and does not take into account that the capacitor bank can actually get depleted, nor does it take into account the current the capacitor bank supplies. (2) on the other hand does not take into account that inductance will cause a zero current at the start, and then rise upwards to the peak value.

The "actual" graph will thus be roughly approximated by following (1) until it meets (2), and then following (2) from then on. And the point where they meet, t_{meet} can be found by:

$$\frac{RCV_0}{L}\left(1 - e^{\frac{-t}{RC}}\right) = \frac{V_0}{R} e^{\frac{-t}{RC}}$$

$$\frac{R^2C}{L}\left(1 - e^{\frac{-t}{RC}}\right) = e^{\frac{-t}{RC}}$$

$$\frac{R^2C}{L} - \frac{R^2C}{L} e^{\frac{-t}{RC}} = e^{\frac{-t}{RC}}$$

$$\frac{R^2C}{L} = e^{\frac{-t}{RC}}\left(1 + \frac{R^2C}{L}\right)$$

$$\frac{\frac{R^2C}{L}}{1 + \frac{R^2C}{L}} = e^{\frac{-t}{RC}}$$

$$\frac{1}{\frac{L}{R^2C} + 1} = e^{\frac{-t}{RC}}$$

$$-\ln\left(\frac{L}{R^2C} + 1\right) = -\frac{t}{RC}$$

$$RC\ln\left(\frac{L}{R^2C} + 1\right) = t = t_{\text{meet}}$$

It must be especially noted that this model is very limited, as it does not take into account that the RLC circuit established is an oscillating one. From online simulations, it appears that the calculation of peak current is quite accurate for normal resistances, i.e. 1-20Ω, and low inductance, i.e. μH, but the t_{meet} is not accurate. Although t_{meet} is at the peak current and predicts it fairly accurately, it seems to also represent the point where the more "real" model starts to diverge significantly from the model proposed here. i.e. it seems that (1) is a good model till t_{meet} is reached.

Otherwise, the two formulas don't really tell us anything more, and we have practically all that is needed for the design of the EMP device.

The last consideration is the wires used to link the capacitor bank and the propagating loop. These carry with them their own inductance as governed by the formula:

$$L = \frac{\mu_0 l}{\pi}\left(\ln\frac{d}{a} + 0.25\right)$$

where l is the length of the wires, d the separation distance, and a the wire radius.
l has to be short, but still need to be long enough for the user to easily maneuver the loop.
d is going to almost be equal to $2a$ due to insulation covering the wires. We would like a to be big because that would mean less resistance contributed by the wires that are not intended to transmit the EMP.

To recapitulate, thus, the design will involve a capacitor bank charged to V which is as high as possible, connected via parallel wires (which separation distance, d, has to be minimum, while being just long enough for convenience, and a for those wires should be as thick as possible) to a **<u>single loop</u>** with a small radius r, and large a.
The EMP will be an oscillating wave, but the greatest effect the EMP will have on its "victim" is during the initial rise, where $\frac{dI}{dt}$ is largest, and after that drop down to zero before going into negative, which is still effective, but because the "reverse pulse" is less steep, the effect is lesser than the rise from 0 to peak.

Power considerations wise, the intention is to design a portable set, hence it should not be bigger than a briefcase or backpack. To estimate the power considerations needed for such a set, we do the following:

Recalling what we established earlier,

$$\varepsilon = \frac{d\Phi}{dt} = \frac{A\mu_0 V}{2rL}$$

But now the total inductance L has to include the parallel wires leading up to the loop as well! Inductors in series contribute to total inductance linearly, and so:

$$\varepsilon = \frac{A\mu_0 V}{2r\left\{\frac{\mu_0 l}{\pi}\left(\ln\frac{d}{a_1} + 0.25\right) + \mu_0 r\left(\ln\frac{8r}{a_2} - 1.75\right)\right\}}$$

$$= \frac{A\mu_0 V}{2r\mu_0\left\{\frac{l}{\pi}\left(\ln\frac{d}{a_1} + 0.25\right) + r\left(\ln\frac{8r}{a_2} - 1.75\right)\right\}}$$

And substituting in the capacitor discharge formula, we get:

$$\varepsilon = \frac{AV_0 e^{\frac{-t}{RC}}}{2r\left\{\frac{l}{\pi}\left(\ln\frac{d}{a_1} + 0.25\right) + r\left(\ln\frac{8r}{a_2} - 1.75\right)\right\}}$$

Typical circuit boards are roughly around iPhone size, which means about 5cm by 10cm. This makes an area of about 50cm^2.

$A = 50\text{cm}^2 = 0.0050\text{m}^2$

If r of the loop we have is about 5cm (which is a nice size to cover any small electronics), then assuming we use the same wire for the loop and the section leading towards it, let both a be 0.2cm = 0.002m, assuming l is 2m which is a comfortable length to play with, d is around $2a$ due to insulation, and as most circuits are concerned, $R < 1$ohm, so let us take it as $1\,\Omega$.

Plugging in all these values:

$$\varepsilon = \frac{0.0050 V_0 e^{\frac{-t}{C}}}{2 \times 0.05 \times \{0.600 + 0.177\}}$$

$$= 0.0644 V_0 e^{\frac{-t}{C}}$$

If we want $\varepsilon \geq 60\text{V}$ which is where most low voltage electronics would break down by, and considering that this is only needed at the start of the EMP burst, then $t = 0$, and the equation essentially becomes:

$$\varepsilon = 0.0644\, V_0 \geq 60\text{V}$$

Solving for that gives a minimum V_0 of around 931V.

From a low voltage regulator IC chip, the datasheet specifies a maximum of 50V for 200ms or less. So assuming that at 200ms, we require 50V, then

$$60e^{\frac{-0.2}{C}} \geq 50$$

$$e^{\frac{-0.2}{C}} \geq \frac{5}{6}$$

$$\frac{-0.2}{C} \geq \ln\frac{5}{6}$$

$$C \leq \frac{-0.2}{\ln\frac{5}{6}}$$

Solving for C gives 1.1F. This is an extremely large value for a capacitor bank, but if the starting voltage is increased, for example to 1000V, then, calculating the same way, the capacitance required becomes significantly reduced, at around 790mF, while giving a higher starting pulse.

The values are still extremely large on the whole. The energy stored inside a capacitor bank of 790mF at 1000V is calculated to be:

$$E = \frac{1}{2}CV^2 = \frac{1}{2}0.79 \times 1000^2$$

= 395kJ

Sure, it’s not in the megajoule range, but it’s still a hefty amount. By comparison, a typical 1.5V alkaline battery stores about 10.8kJ of energy. One shot of this EMP is going to consume 40 of those batteries!

Conclusion:

Thus from all the formulas, we can see that a whole range of factors affect the production of an EMP to fry whatever electronics we wish to. But mainly, the objective should be to create a capacitor bank with large voltage (so as to reduce the effect that resistance has on limiting the peak current), with a large enough capacitance so that the pulse lasts long enough, but yet not too large to be too heavy. The discharge should be through a single loop of radius just small enough to cover the entire “victim”, while being as thick as possible. The wires leading up to the discharge should be put exactly side by side, i.e. parallel, and be thick to reduce inductance and resistance. If some of the values are adjusted, it may be possible to reduce the energy required, and thus make the EMP more efficient and more effective.

Common sense also tells us to shield all other parts of the circuit except the loop so that EMP is only radiated out where needed, to avoid collateral damage. Electrical insulation is also needed to protect against the high voltages.